# CS 412 Intro. to Data Mining

## Chapter 2. Getting to Know Your Data

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# Chapter 2. Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

# Types of Data Sets: (1) Record Data

- Relational records
  - Relational tables, highly structured
- Data matrix, e.g., numerical matrix, crosstabs

| | China | England | France | Japan | USA | Total |
|---|---|---|---|---|---|---|
| Active Outdoors Crochet Glove | | 12.00 | 4.00 | 1.00 | 240.00 | 257.00 |
| Active Outdoors Lycra Glove | | 10.00 | 6.00 | | 323.00 | 339.00 |
| InFlux Crochet Glove | 3.00 | 6.00 | 8.00 | | 132.00 | 149.00 |
| InFlux Lycra Glove | | 2.00 | | | 143.00 | 145.00 |
| Triumph Pro Helmet | 3.00 | 1.00 | 7.00 | | 333.00 | 344.00 |
| Triumph Vertigo Helmet | | 3.00 | 22.00 | | 474.00 | 499.00 |
| Xtreme Adult Helmet | 8.00 | 8.00 | 7.00 | 2.00 | 251.00 | 276.00 |
| Xtreme Youth Helmet | | 1.00 | | | 76.00 | 77.00 |
| Total | 14.00 | 43.00 | 54.00 | 3.00 | 1,972.00 | 2,086.00 |

Person:

| Pers_ID | Surname | First_Name | City |
|---|---|---|---|
| 0 | Miller | Paul | London |
| 1 | Ortega | Alvaro | Valencia |
| 2 | Huber | Urs | Zurich |
| 3 | Blanc | Gaston | Paris |
| 4 | Bertolini | Fabrizio | Rom |

no relation

Car:

| Car_ID | Model | Year | Value | Pers_ID |
|---|---|---|---|---|
| 101 | Bentley | 1973 | 100000 | 0 |
| 102 | Rolls Royce | 1965 | 330000 | 0 |
| 103 | Peugeot | 1993 | 500 | 3 |
| 104 | Ferrari | 2005 | 150000 | 4 |
| 105 | Renault | 1998 | 2000 | 3 |
| 106 | Renault | 2001 | 7000 | 3 |
| 107 | Smart | 1999 | 2000 | 2 |

- Transaction data

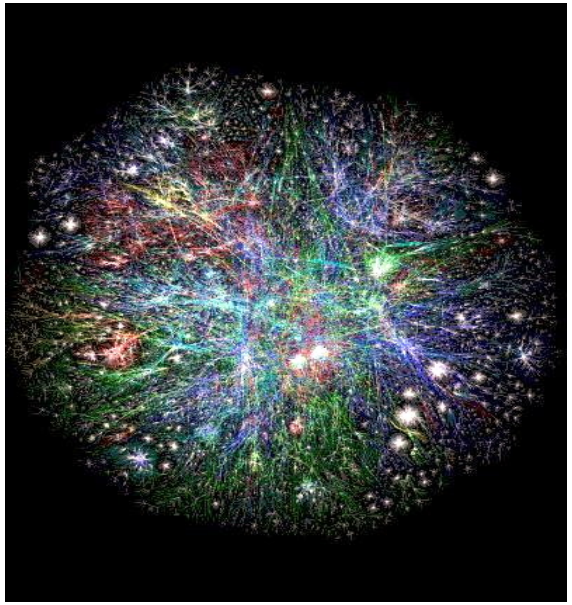
| *TID* | *Items* |
|---|---|
| 1 | Bread, Coke, Milk |
| 2 | Beer, Bread |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Coke, Diaper, Milk |

| | team | coach | play | ball | score | game | win | lost | timeout | season |
|---|---|---|---|---|---|---|---|---|---|---|
| Document 1 | 3 | 0 | 5 | 0 | 2 | 6 | 0 | 2 | 0 | 2 |
| Document 2 | 0 | 7 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 0 |
| Document 3 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 3 | 0 |

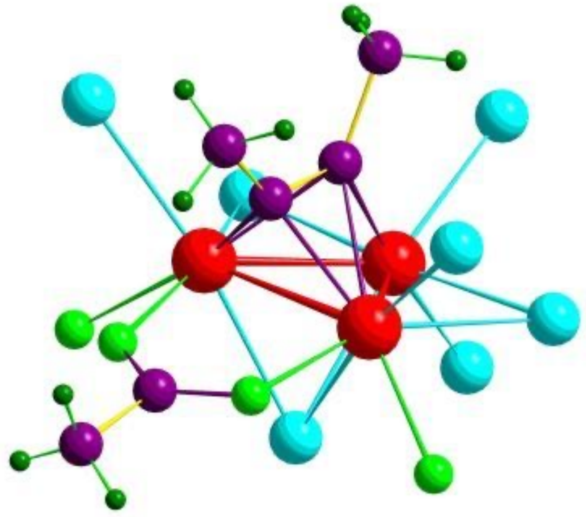
- Document data: Term-frequency vector (matrix) of text documents

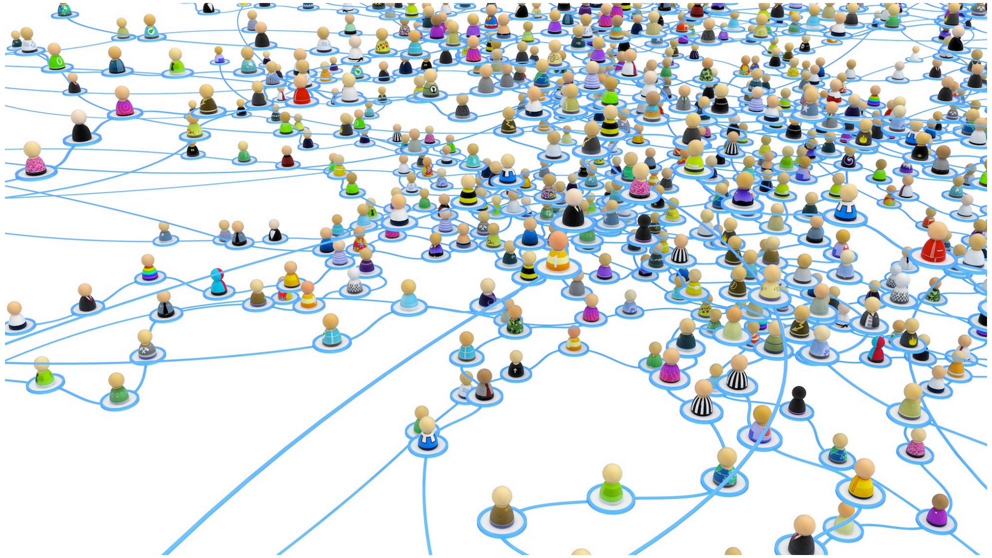
# Types of Data Sets: (2) Graphs and Networks

- Transportation network



- World Wide Web



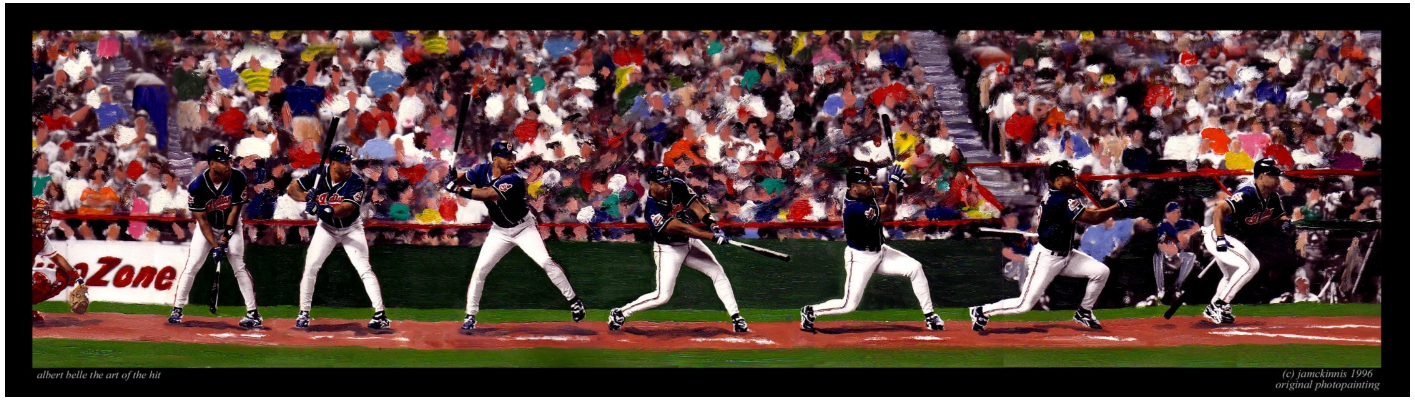


- Molecular Structures

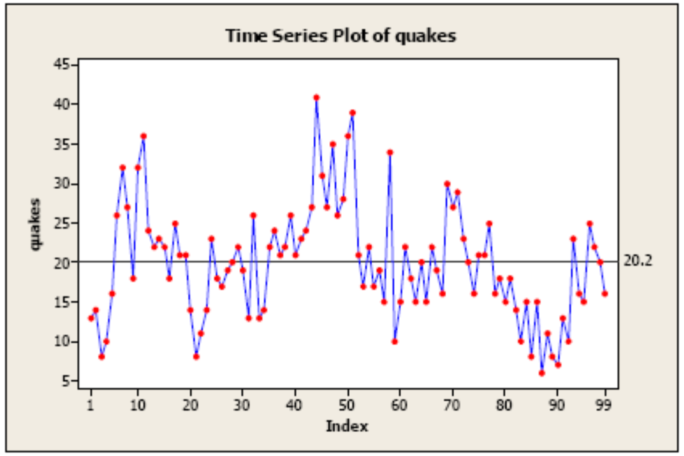
- Social or information networks

# Types of Data Sets: (3) Ordered Data

- Video data: sequence of images




- Temporal data: time-series



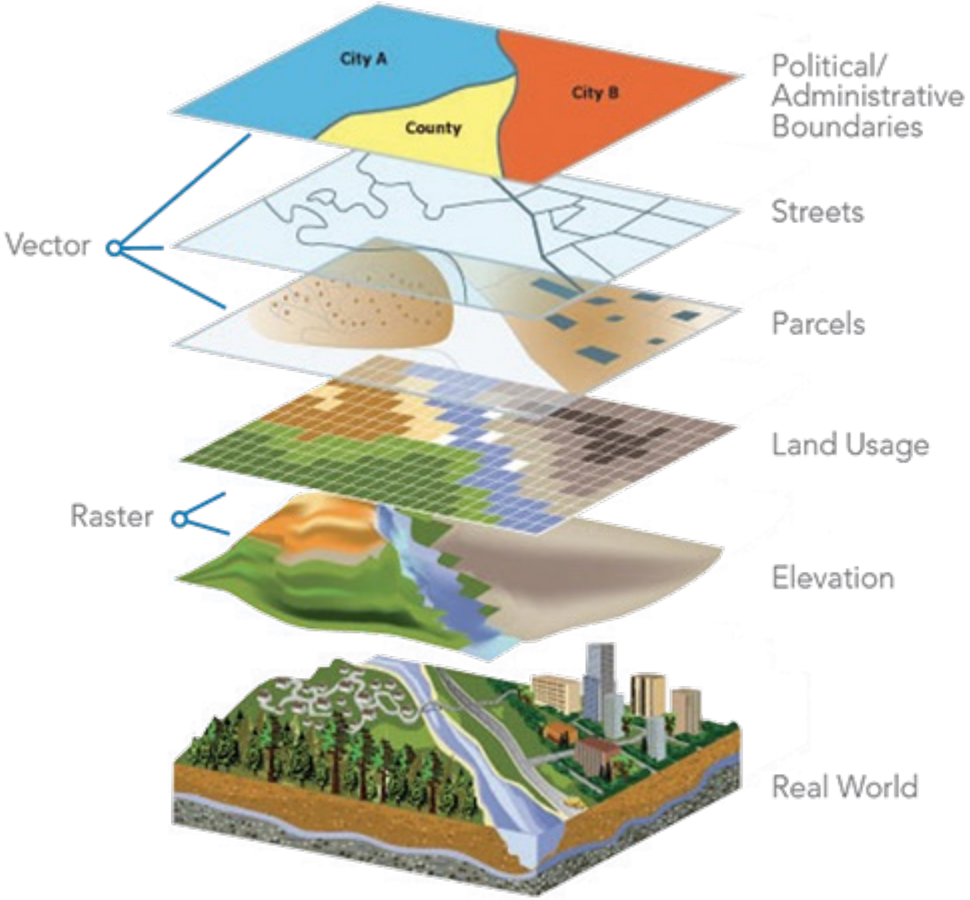

- Sequential Data: transaction sequences

- Genetic sequence data

# Types of Data Sets: (4) Spatial, image and multimedia Data

- Spatial data: maps

- Image data:



# Important Characteristics of Structured Data

- Dimensionality
  - Curse of dimensionality
- Sparsity
  - Only presence counts
- Resolution
  - Patterns depend on the scale
- Distribution
  - Centrality and dispersion

# Data Objects

- Data sets are made up of data objects
- A **data object** represents an entity
- Examples:
  - sales database:  customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples , examples, instances, data points, objects, tuples*
- Data objects are described by **attributes**
- Database rows → data objects; columns → attributes

# Attributes

- **Attribute (**or **dimensions, features, variables**)
  - A data field, representing a characteristic or feature of a data object.
  - *E.g., customer _ID, name, address*
- Types:
  - Nominal (e.g., red, blue)
  - Binary (e.g., {true, false})
  - Ordinal (e.g., {freshman, sophomore, junior, senior})
  - Numeric: quantitative
    - Interval-scaled: 100°C is interval scales
    - Ratio-scaled: 100°K is ratio scaled since it is twice as high as 50 °K
- Q1: Is student ID a nominal, ordinal, or interval-scaled data?
- Q2: What about eye color? Or color in the color spectrum of physics?

# Attribute Types

- **Nominal:** categories, states, or "names of things"
  - *Hair_color* = {*auburn, black, blond, brown, grey, red, white*}
  - marital status, occupation, ID numbers, zip codes
- **Binary**
  - Nominal attribute with only 2 states (0 and 1)
  - Symmetric binary: both outcomes equally important
    - e.g., gender
  - Asymmetric binary: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known
  - *Size* = {*small, medium, large*}, grades, army rankings

# Numeric Attribute Types

- Quantity (integer or real-valued)
- **Interval**
  - Measured on a scale of **equal-sized units**
  - Values have order
    - E.g., *temperature in C˚or F˚, calendar dates*
  - No true zero-point
- **Ratio**
  - Inherent **zero-point**
  - We can speak of values as being an order of magnitude larger than the unit of measurement (10 K˚ is twice as high as 5 K˚).
    - e.g., *temperature in Kelvin, length, counts, monetary quantities*

# Discrete vs. Continuous Attributes

- **Discrete Attribute**
  - Has only a finite or countably infinite set of values
    - E.g., zip codes, profession, or the set of words in a collection of documents
  - Sometimes, represented as integer variables
  - Note: Binary attributes are a special case of discrete attributes
- **Continuous Attribute**
  - Has real numbers as attribute values
    - E.g., temperature, height, or weight
  - Practically, real values can only be measured and represented using a finite number of digits
  - Continuous attributes are typically represented as floating-point variables

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- Data Visualization
- Measuring Data Similarity and Dissimilarity
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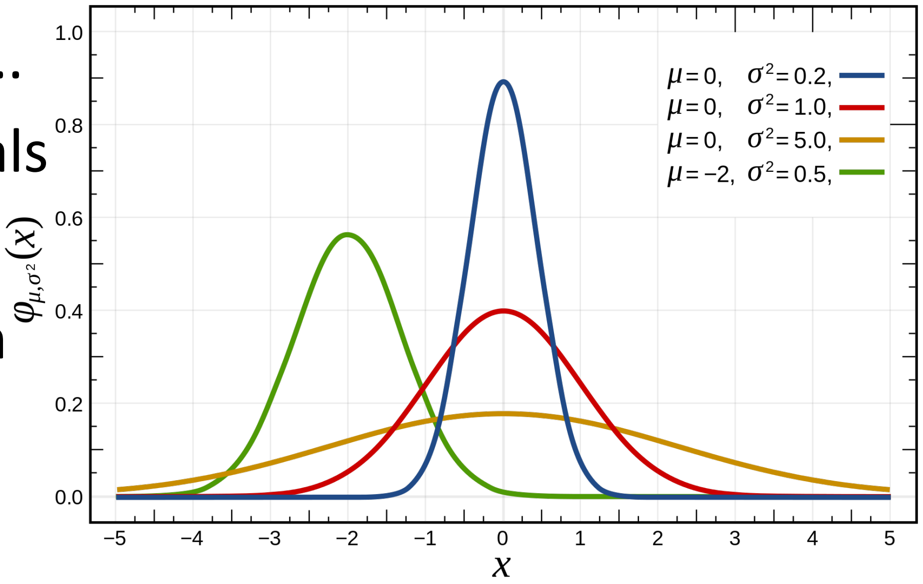
# Basic Statistical Descriptions of Data

- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - Median, max, min, quantiles, outliers, variance, ...
- Numerical dimensions correspond to sorted intervals
  - Data dispersion:
    - Analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals
- Dispersion analysis on computed measures
  - Folding measures into numerical dimensions
  - Boxplot or quantile analysis on the transformed cube



# Measuring the Central Tendency: (1) Mean

- Mean (algebraic measure) (sample vs. population):

  Note: *n* is sample size and *N* is population size.

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \mu = \frac{\sum x}{N}$$

- Weighted arithmetic mean:

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

- Trimmed mean:
  - Chopping extreme values (e.g., Olympics gymnastics score computation)

# Measuring the Central Tendency: (2) Median

- Median:
  - Middle value if odd number of values, or average of the middle two values otherwise
- Estimated by interpolation (for *grouped data*):

| *age* | *frequency* |
|---|---|
| 1–5 | 200 |
| 6–15 | 450 |
| 16–20 | 300 |
| 21–50 | 1500 |
| 51–80 | 700 |
| 81–110 | 44 |

Approximate median

Sum before the median interval

Interval width ($L_2 - L_1$)

$$median = L_1 + \left(\frac{n/2 - (\sum freq)_l}{freq_{median}}\right) width$$

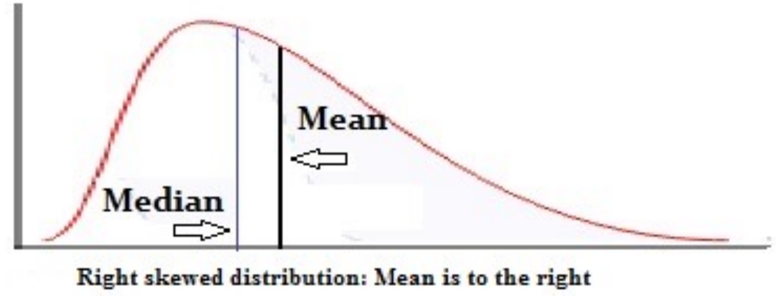
Low interval limit

# Measuring the Central Tendency: (3) Mode

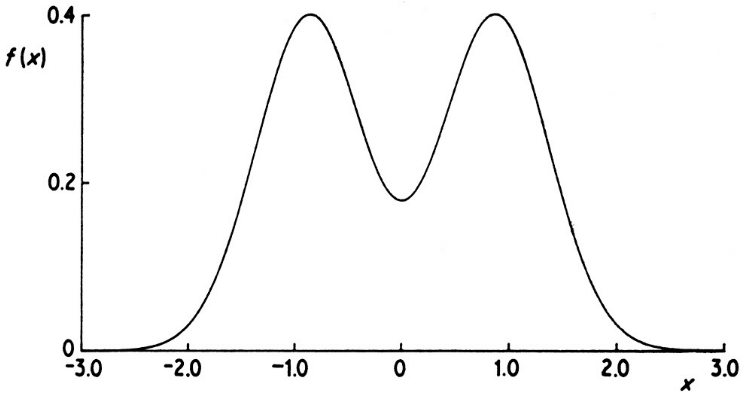
- Mode: Value that occurs most frequently in the data

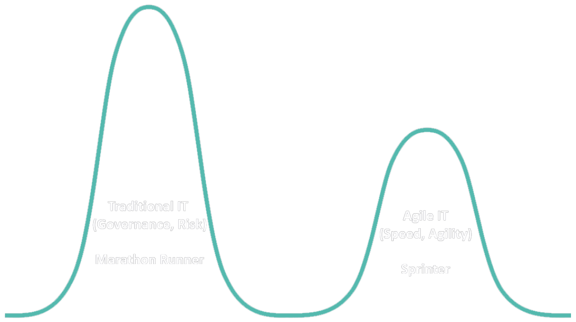
- Unimodal
  - Empirical formula:

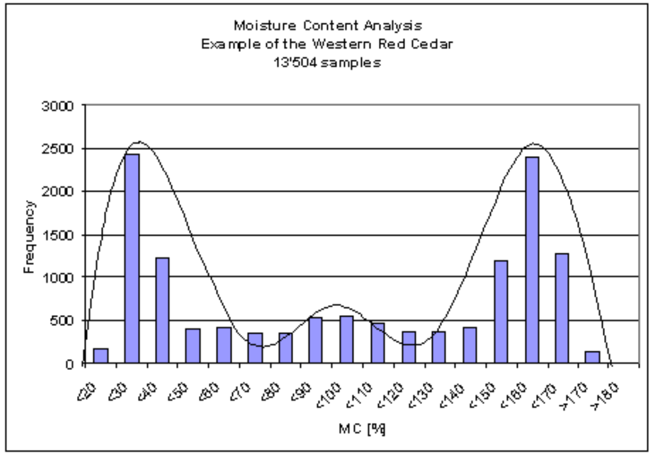
$$mean - mode = 3 \times (mean - median)$$




- Multi-modal
  - Bimodal
  - Trimodal

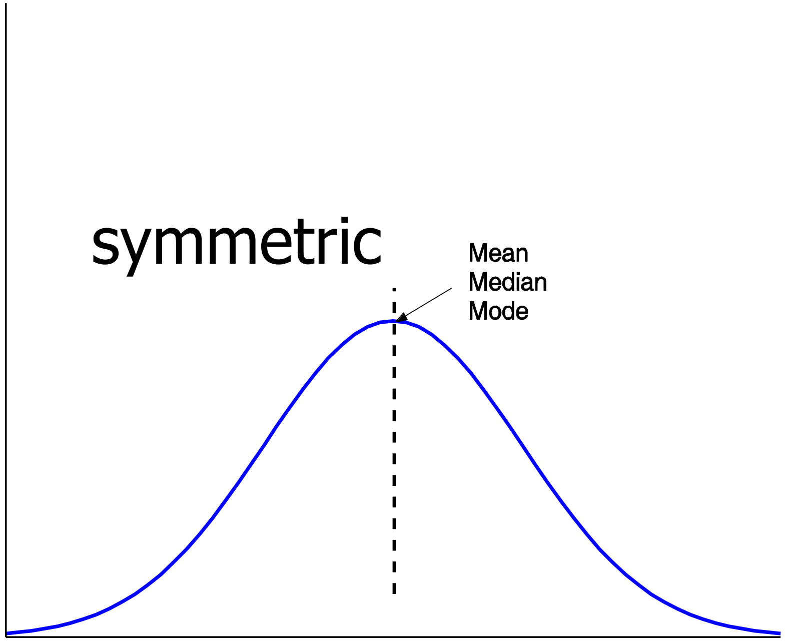


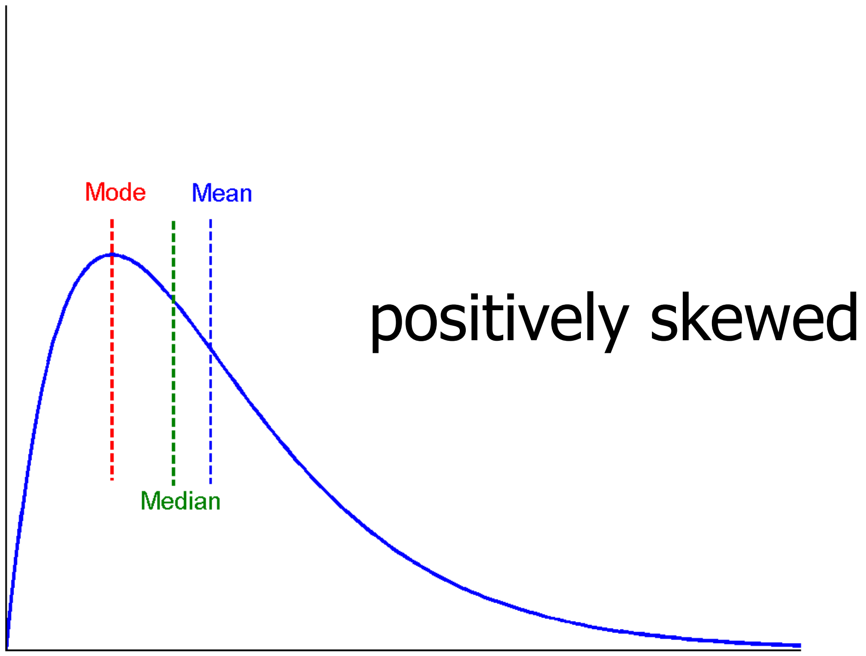



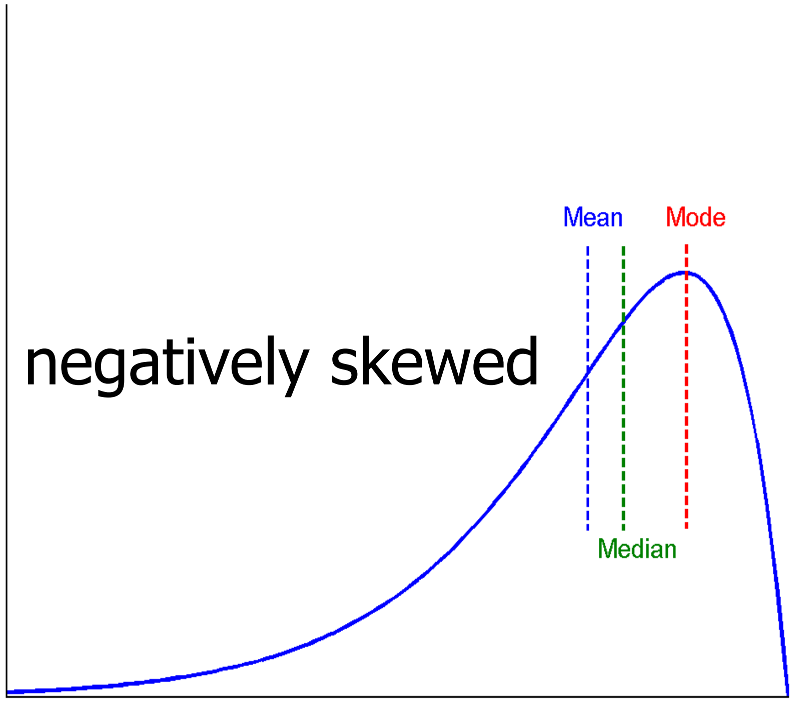


# Symmetric vs. Skewed Data

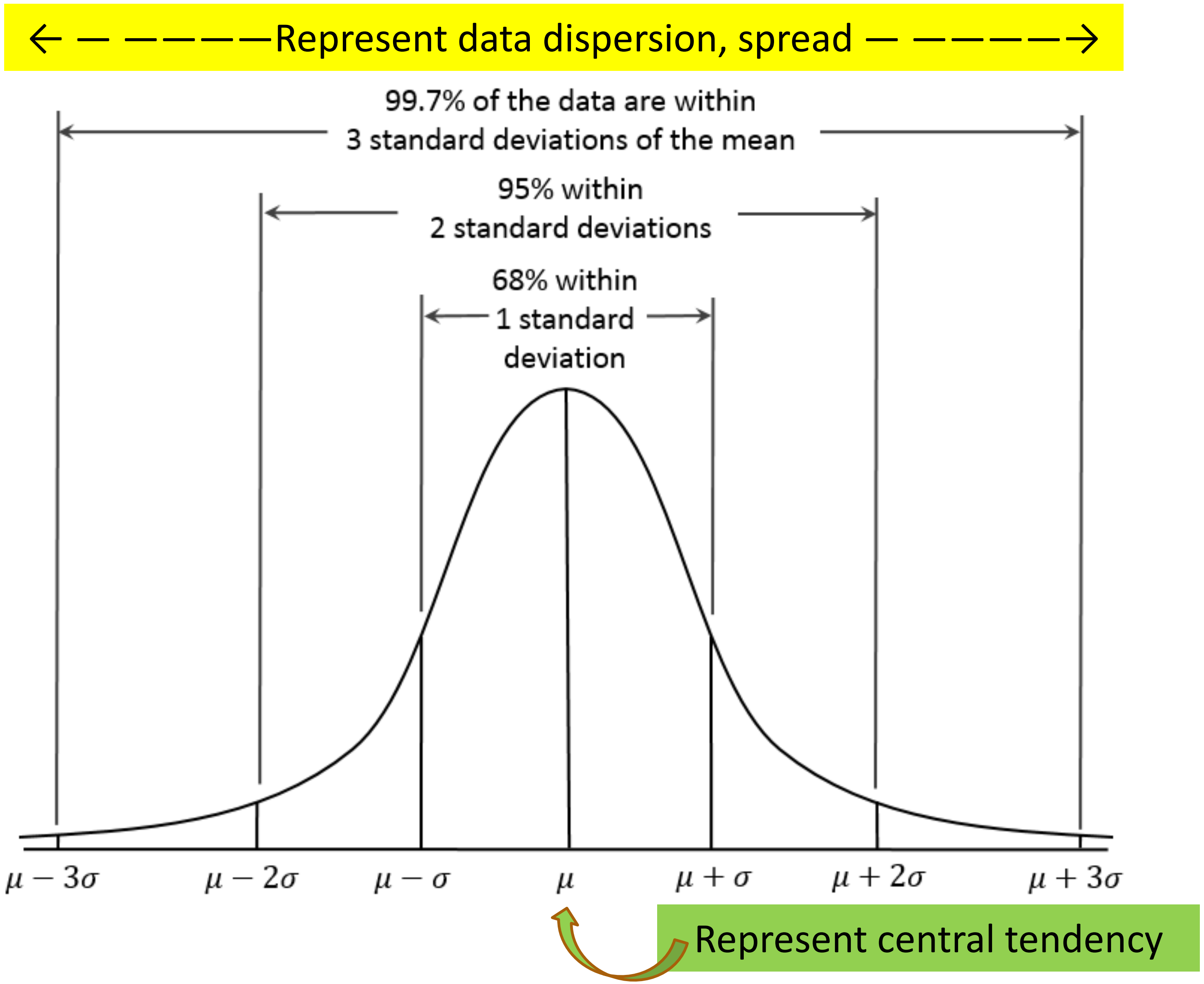
- Median, mean and mode of symmetric, positively and negatively skewed data



# Properties of Normal Distribution Curve



# Measures Data Distribution: Variance and Standard Deviation

- Variance and standard deviation (*sample: s, population: σ*)
  - **Variance**: (algebraic, scalable computation)
    - Q: Can you compute it incrementally and efficiently?

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}[\sum_{i=1}^{n}x_i^2 - \frac{1}{n}(\sum_{i=1}^{n}x_i)^2]$$

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n}(x_i - \mu)^2 = \frac{1}{N}\sum_{i=1}^{n}x_i^2 - \mu^2$$

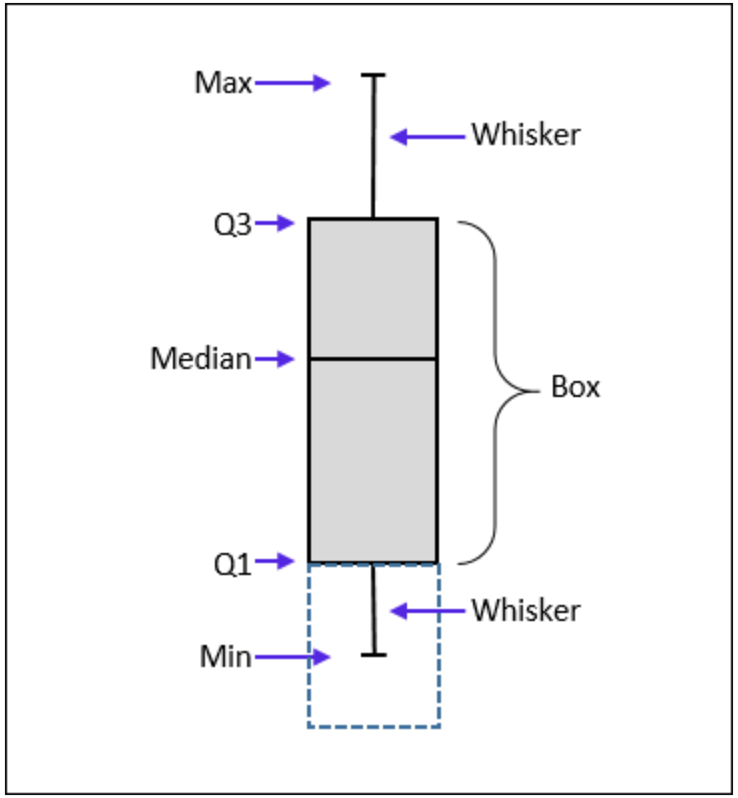
  - **Standard deviation** *s (or σ)* is the square root of variance $s^2$ (*or* $\sigma^2$)

# Graphic Displays of Basic Statistical Descriptions

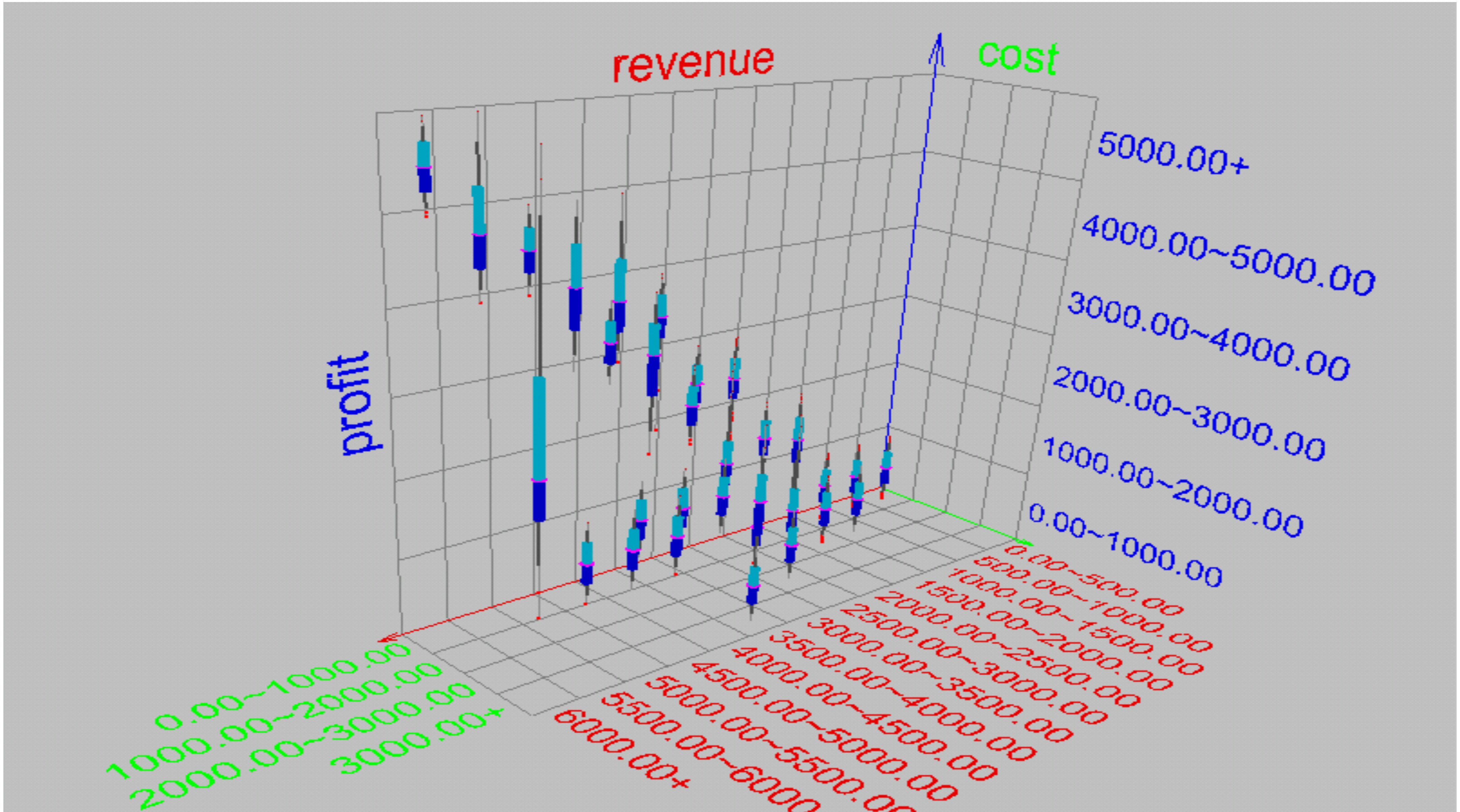
- **Boxplot**: graphic display of five-number summary
- **Histogram**: x-axis are values, y-axis repres. frequencies
- **Quantile plot**: each value $x_i$ is paired with $f_i$ indicating that approximately 100 $f_i$ % of data are $\leq x_i$
- **Quantile-quantile (q-q) plot**: graphs the quantiles of one univariant distribution against the corresponding quantiles of another
- **Scatter plot**: each pair of values is a pair of coordinates and plotted as points in the plane

# Measuring the Dispersion of Data: Quartiles & Boxplots

- **Quartiles**: $Q_1$ (25th percentile), $Q_3$ (75th percentile)
- **Inter-quartile range**: IQR = $Q_3 - Q_1$
- **Five number summary**: min, $Q_1$, median, $Q_3$, max
- **Boxplot**: Data is represented with a box
  - $Q_1$, $Q_3$, IQR: The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
  - Median ($Q_2$) is marked by a line within the box
  - Whiskers: two lines outside the box extended to Minimum and Maximum
  - Outliers: points beyond a specified outlier threshold, plotted individually
    - **Outlier**: usually, a value higher/lower than 1.5 x IQR



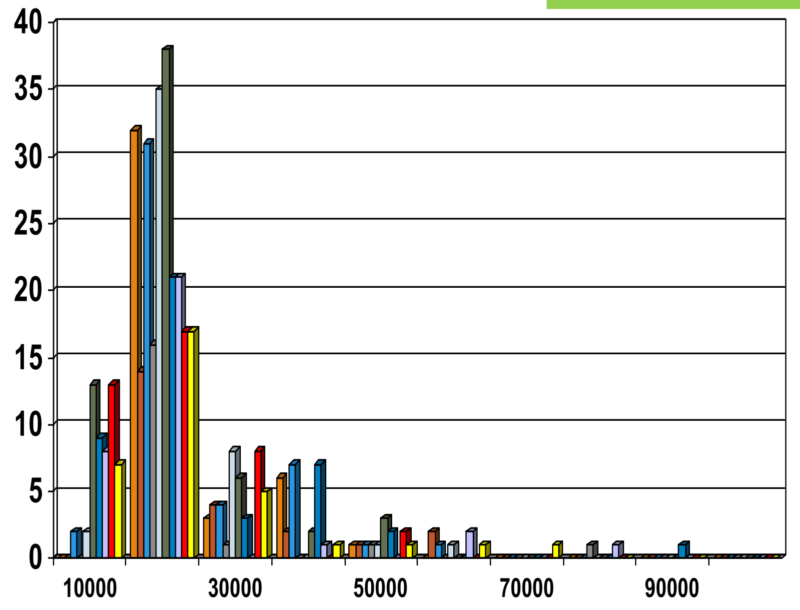
# Visualization of Data Dispersion: 3-D Boxplots

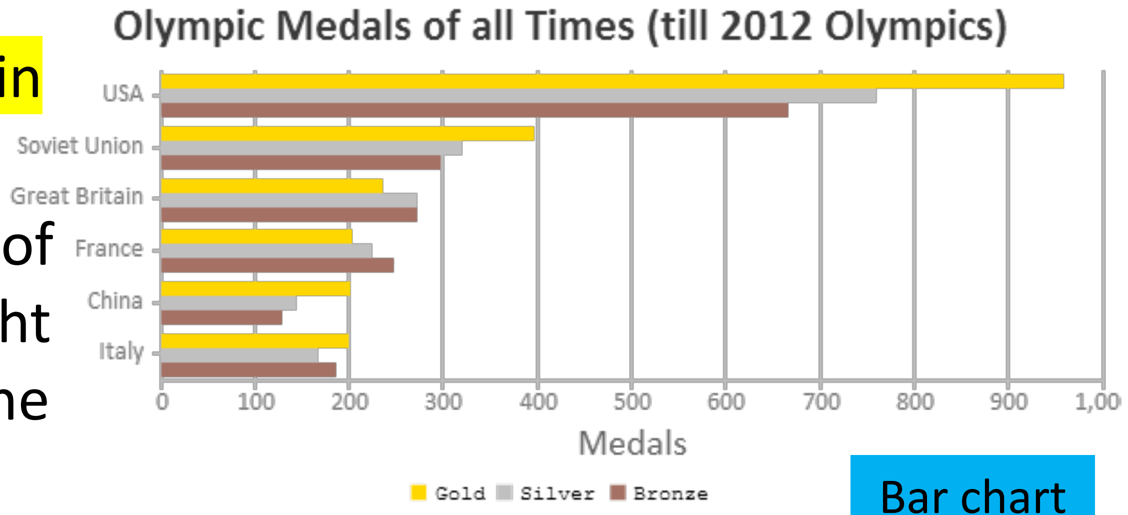


# Histogram Analysis

- Histogram: Graph display of tabulated frequencies, shown as bars
- Differences between histograms and bar charts
  - Histograms are used to show distributions of variables while bar charts are used to compare variables
  - Histograms plot binned quantitative data while bar charts plot categorical data
  - Bars can be reordered in bar charts but not in histograms
  - Differs from a bar chart in that it is the area of the bar that denotes the value, not the height as in bar charts, a crucial distinction when the categories are not of uniform width

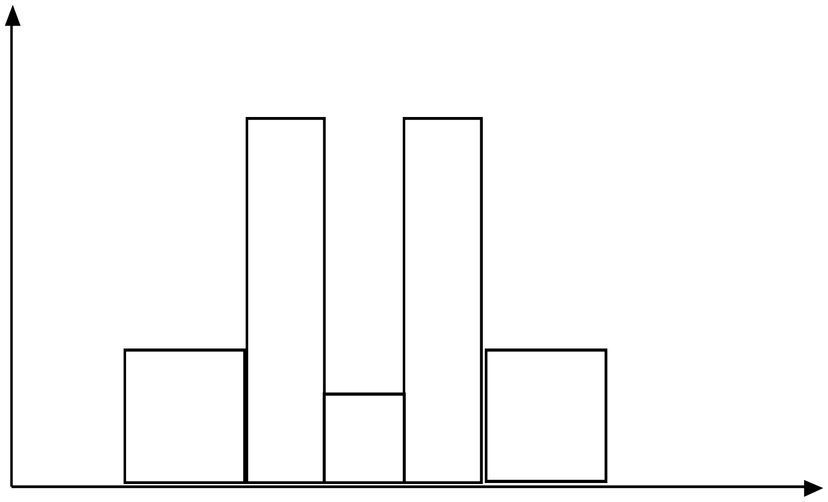
Histogram

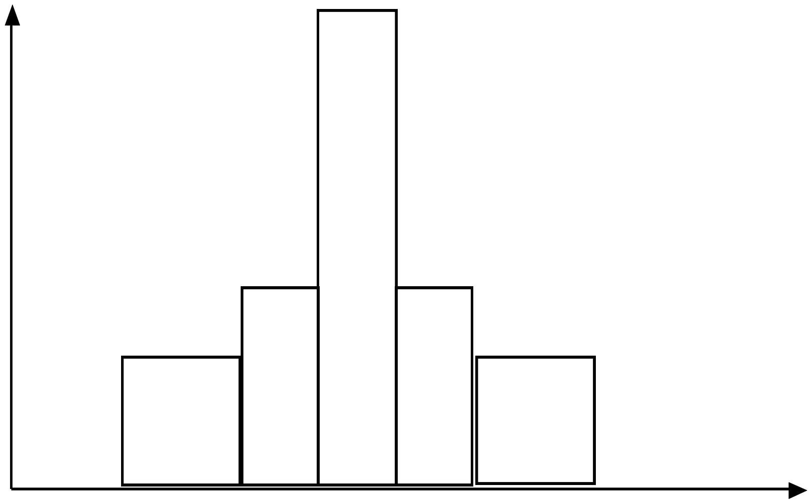



Bar chart

# Histograms Often Tell More than Boxplots

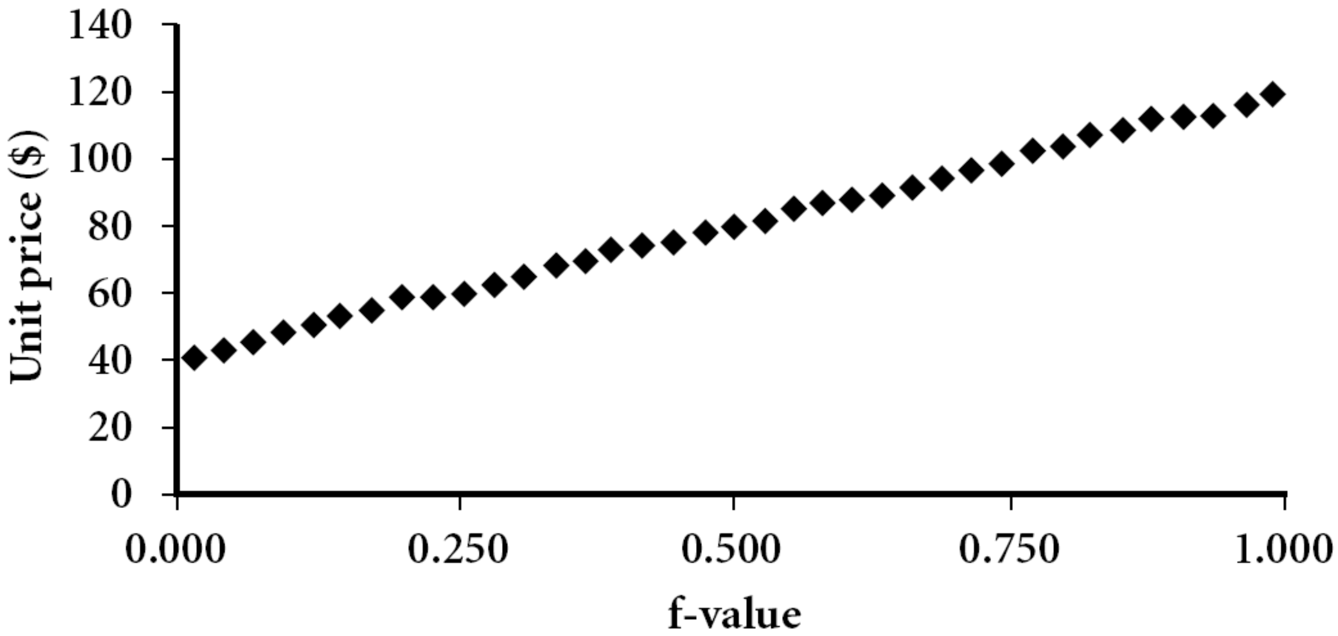




- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

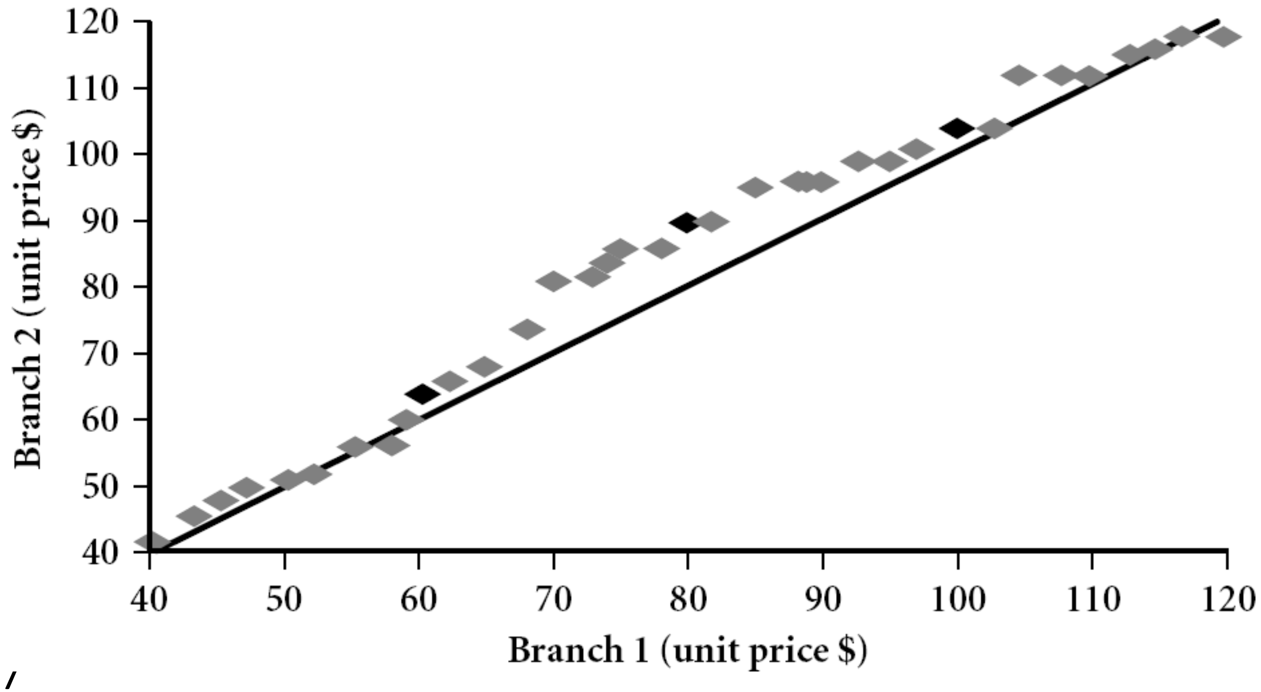
# Quantile Plot

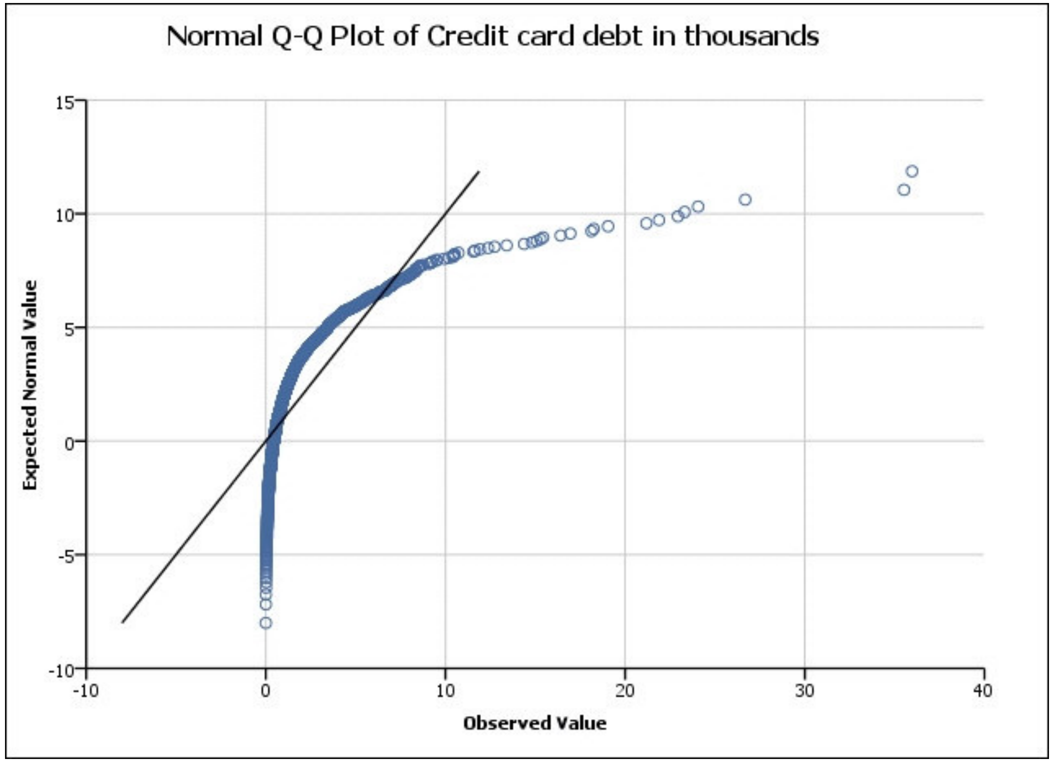
- Displays all of the data (allowing the user to assess both the overall behavior and unusual occurrences)
- Plots **quantile** information
  - For a data $x_i$ data sorted in increasing order, $f_i$ indicates that approximately 100 $f_i$% of the data are below or equal to the value $x_i$



# Quantile-Quantile (Q-Q) Plot

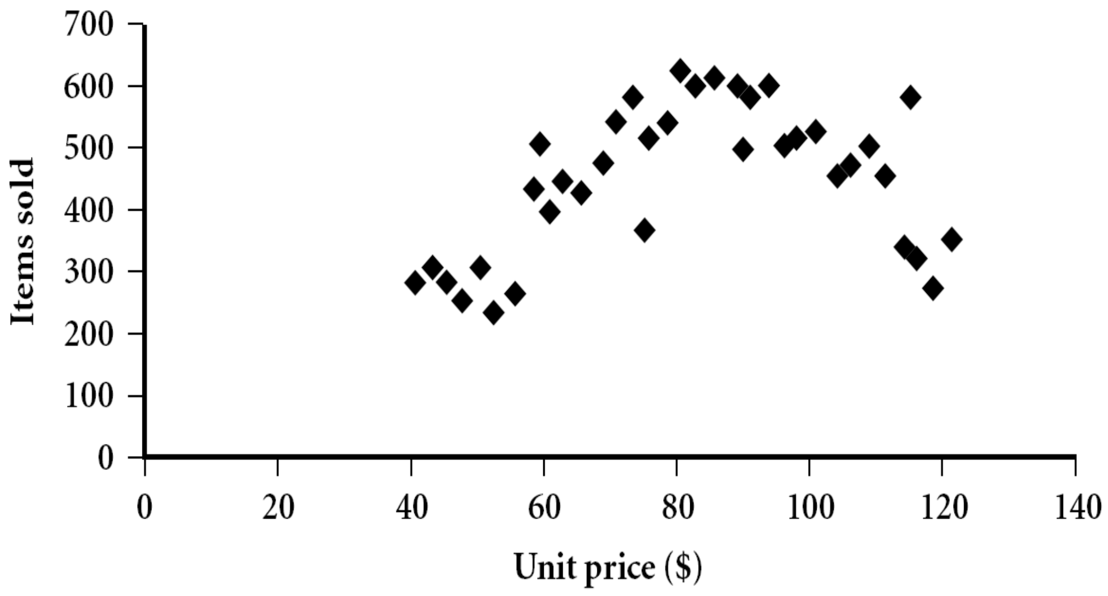
- Graphs the quantiles of one univariate distribution against the corresponding quantiles of another
- View: Is there is a shift in going from one distribution to another?
- Example shows unit price of items sold at Branch 1 vs. Branch 2 for each quantile. Unit prices of items sold at Branch 1 tend to be lower than those at Branch 2

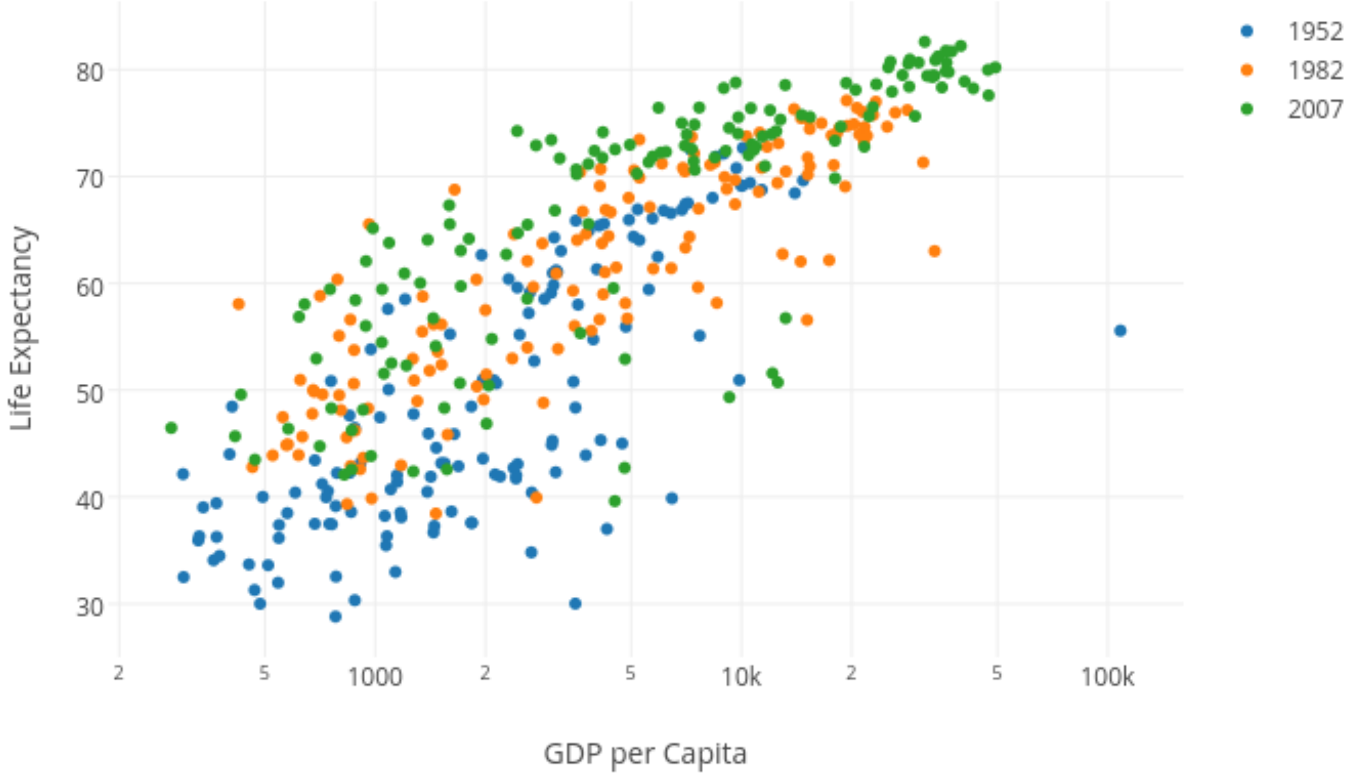


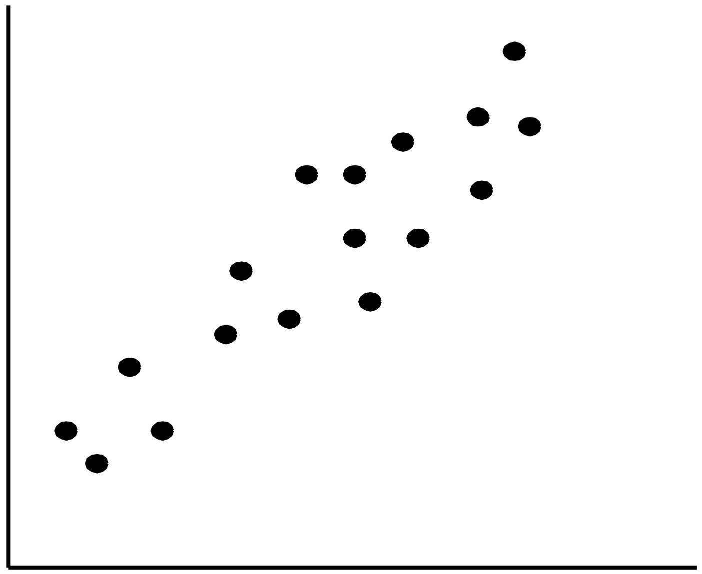
# Scatter plot

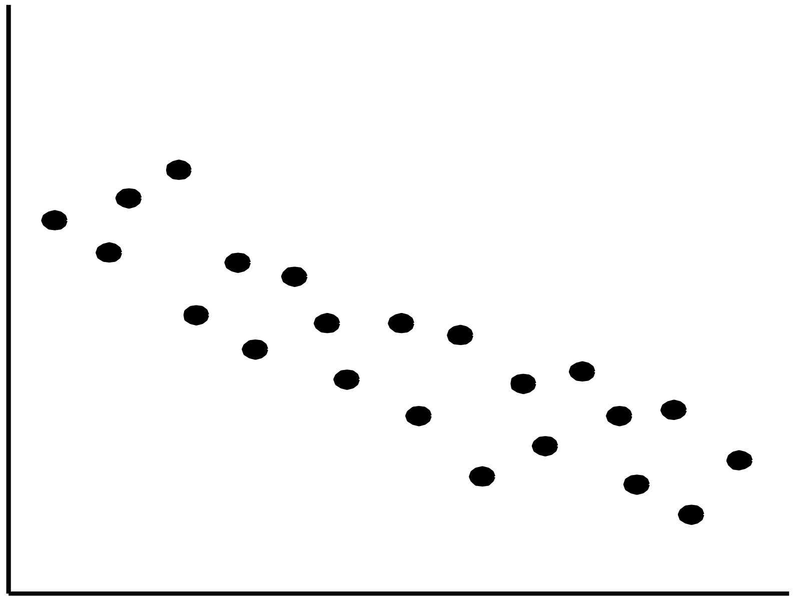
- Provides a first look at bivariate data to see clusters of points, outliers, etc.
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane

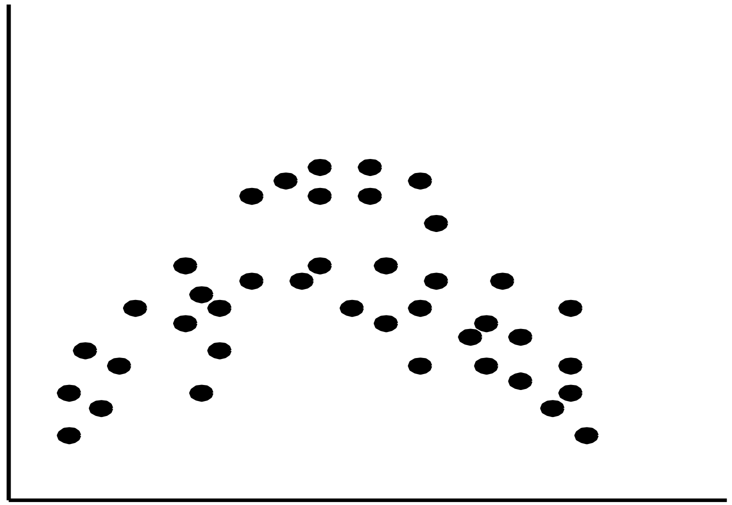


# Positively and Negatively Correlated Data

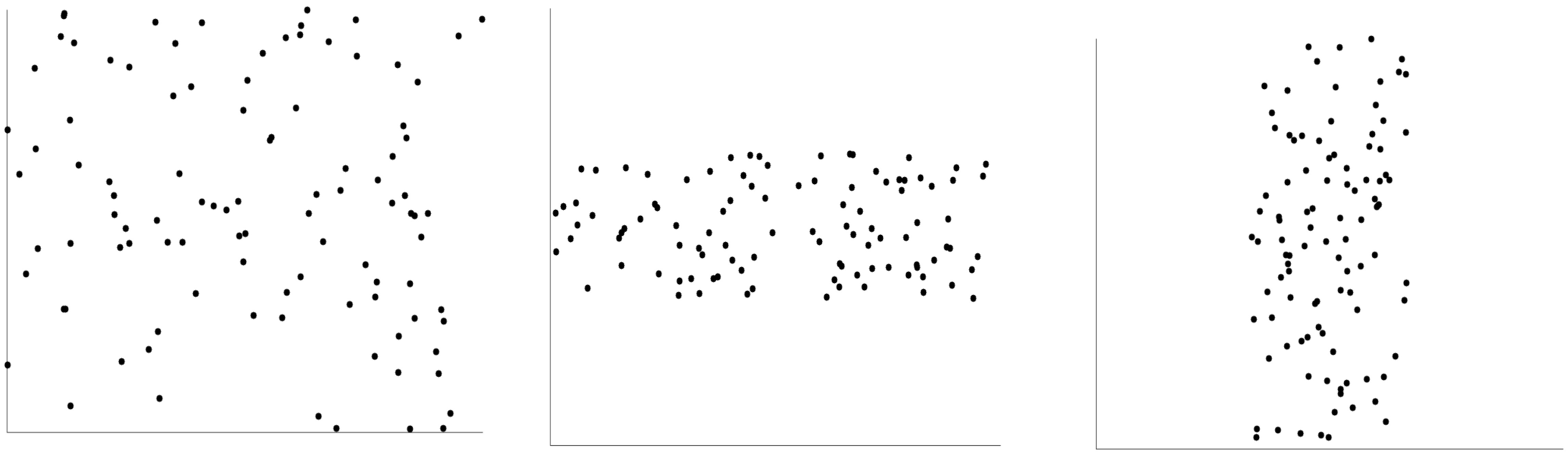






- The left half fragment is positively correlated
- The right half is negative correlated

# Uncorrelated Data



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# Standardizing Numeric Data

- Z-score: $z = \frac{x - \mu}{\sigma}$
  - X: raw score to be standardized, μ: mean of the population, σ: standard deviation
  - the distance between the raw score and the population mean in units of the standard deviation
  - negative when the raw score is below the mean, "+" when above
- An alternative way: Calculate the mean absolute deviation

$$s_f = \frac{1}{n}(|x_{1f} - m_f| + |x_{2f} - m_f| + ... + |x_{nf} - m_f|)$$

where

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + ... + x_{nf}).$$

  - standardized measure (*z-score*): $z_{if} = \frac{x_{if} - m_f}{s_f}$
- Using mean absolute deviation is more robust than using standard deviation

# Similarity, Dissimilarity, and Proximity

- **Similarity measure** or **similarity function**
  - A real-valued function that quantifies the similarity between two objects
  - Measure how two data objects are alike: The higher value, the more alike
  - Often falls in the range [0,1]:  0: no similarity; 1: completely similar
- **Dissimilarity** (or **distance**) measure
  - Numerical measure of how different two data objects are
  - In some sense, the inverse of similarity:  The lower, the more alike
  - Minimum dissimilarity is often 0 (i.e., completely similar)
  - Range [0, 1] or [0, ∞) , depending on the definition
- **Proximity** usually refers to either similarity or dissimilarity

# Data Matrix and Dissimilarity Matrix

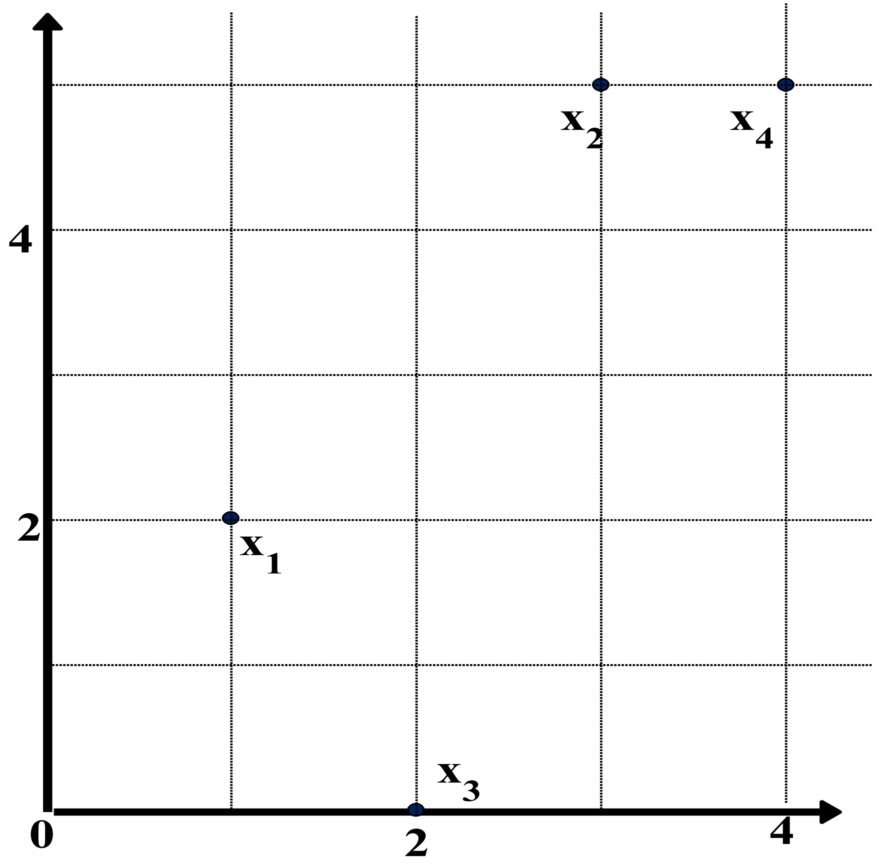
- Data matrix
  - A data matrix of n data points with *l* dimensions

$$D = \begin{pmatrix} x_{11} & x_{12} & \ldots & x_{1l} \\ x_{21} & x_{22} & \ldots & x_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \ldots & x_{nl} \end{pmatrix}$$

- Dissimilarity (distance) matrix
  - n data points, but registers only the distance *d*(*i, j*) (typically metric)
  - Usually symmetric, thus a triangular matrix
  - Distance functions are usually different for real, boolean, categorical, ordinal, ratio, and vector variables
  - Weights can be associated with different variables based on applications and data semantics

$$\begin{pmatrix} 0 & & & \\ d(2,1) & 0 & & \\ \vdots & \vdots & \ddots & \\ d(n,1) & d(n,2) & \ldots & 0 \end{pmatrix}$$

# Example: Data Matrix and Dissimilarity Matrix




**Data Matrix**

| point | attribute1 | attribute2 |
|---|---|---|
| *x1* | 1 | 2 |
| *x2* | 3 | 5 |
| *x3* | 2 | 0 |
| *x4* | 4 | 5 |

**Dissimilarity Matrix (by Euclidean Distance)**

| | *x1* | *x2* | *x3* | *x4* |
|---|---|---|---|---|
| *x1* | 0 | | | |
| *x2* | 3.61 | 0 | | |
| *x3* | 2.24 | 5.1 | 0 | |
| *x4* | 4.24 | 1 | 5.39 | 0 |

# Distance on Numeric Data: Minkowski Distance

- Minkowski distance: A popular distance measure

$$d(i,j) = \sqrt[p]{|x_{i1} - x_{j1}|^p + |x_{i2} - x_{j2}|^p + \cdots + |x_{il} - x_{jl}|^p}$$

  where $i = (x_{i1}, x_{i2}, \ldots, x_{il})$ and $j = (x_{j1}, x_{j2}, \ldots, x_{jl})$ are two $l$-dimensional data objects, and $p$ is the order (the distance so defined is also called L-$p$ norm)

- Properties
  - d(i, j) > 0 if i ≠ j, and d(i, i) = 0 (Positivity)
  - d(i, j) = d(j, i) (Symmetry)
  - d(i, j) ≤ d(i, k) + d(k, j) (Triangle Inequality)
- A distance that satisfies these properties is a metric
- Note: There are nonmetric dissimilarities, e.g., set differences

# Special Cases of Minkowski Distance

- $p$ = 1: ($L_1$ norm) Manhattan (or city block) distance
  - E.g., the Hamming distance: the number of bits that are different between two binary vectors

$$d(i,j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \cdots + |x_{il} - x_{jl}|$$

- $p$ = 2: ($L_2$ norm) Euclidean distance

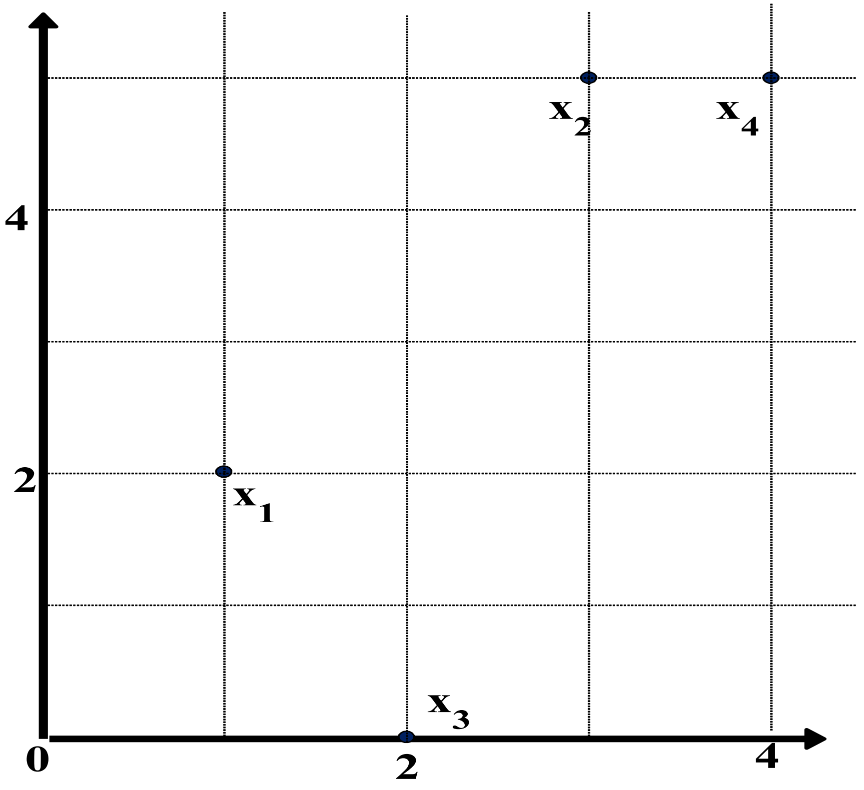
$$d(i,j) = \sqrt{|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \cdots + |x_{il} - x_{jl}|^2}$$

- $p \to \infty$: ($L_{max}$ norm, $L_\infty$ norm) "supremum" distance
  - The maximum difference between any component (attribute) of the vectors

$$d(i,j) = \lim_{p\to\infty} \sqrt[p]{|x_{i1} - x_{j1}|^p + |x_{i2} - x_{j2}|^p + \cdots + |x_{il} - x_{jl}|^p} = \max_{f=1}^{l} |x_{if} - x_{jf}|$$

# Example: Minkowski Distance at Special Cases

| point | attribute 1 | attribute 2 |
|---|---|---|
| x1 | 1 | 2 |
| x2 | 3 | 5 |
| x3 | 2 | 0 |
| x4 | 4 | 5 |




## Manhattan ($L_1$)

| L | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 5 | 0 | | |
| x3 | 3 | 6 | 0 | |
| x4 | 6 | 1 | 7 | 0 |

## Euclidean ($L_2$)

| L2 | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 3.61 | 0 | | |
| x3 | 2.24 | 5.1 | 0 | |
| x4 | 4.24 | 1 | 5.39 | 0 |

## Supremum ($L_\infty$)

| $L_\infty$ | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 3 | 0 | | |
| x3 | 2 | 5 | 0 | |
| x4 | 3 | 1 | 5 | 0 |

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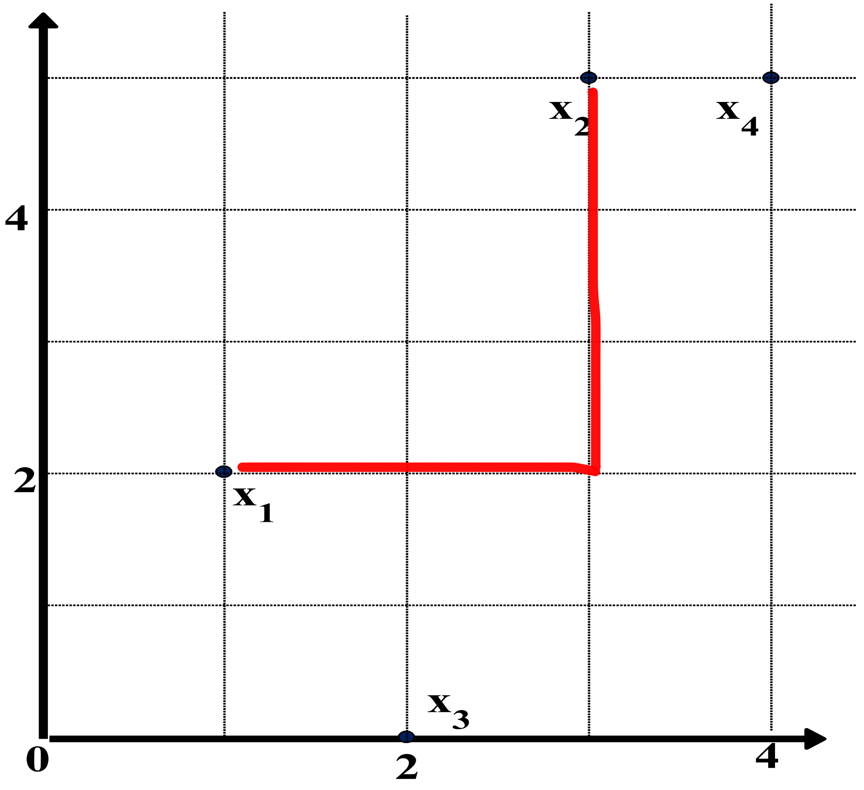


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| L | x1 | x2 | x3 | x4 |
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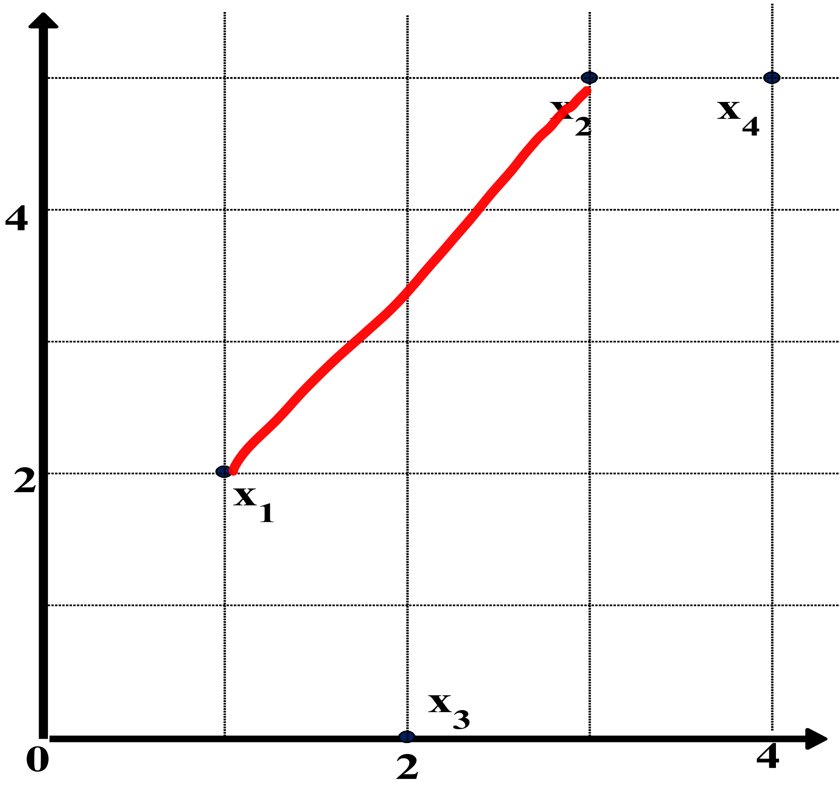


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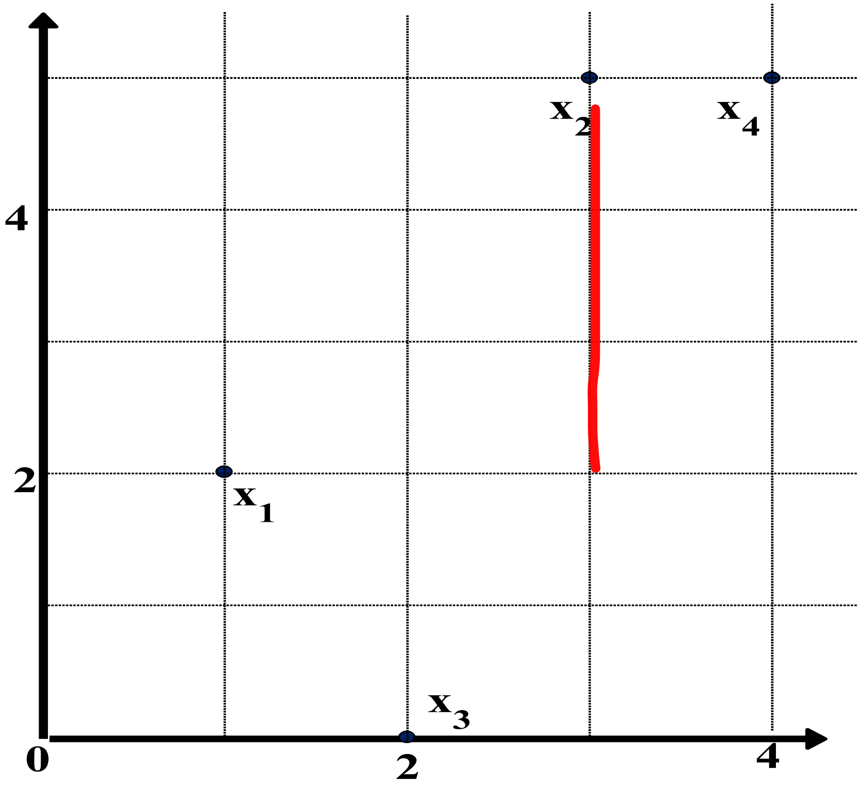
| x3 | 2 | 0 |
| x4 | 4 | 5 |




## Manhattan ($L_1$)

| L | x1 | x2 | x3 | x4 |
|---|---|---|---|---|
| x1 | 0 | | | |
| x2 | 5 | 0 | | |
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| x4 | 6 | 1 | 7 | 0 |

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| x2 | 3 | 0 | | |
| x3 | 2 | 5 | 0 | |
| x4 | 3 | 1 | 5 | 0 |

# Proximity Measure for Binary Attributes

- A contingency table for binary data

| | | Object $j$ | | |
|---|---|---|---|---|
| | | 1 | 0 | sum |
| Object $i$ | 1 | $q$ | $r$ | $q+r$ |
| | 0 | $s$ | $t$ | $s+t$ |
| | sum | $q+s$ | $r+t$ | $p$ |

- Distance measure for symmetric binary variables
$$d(i, j) = \frac{r+s}{q+r+s+t}$$
- Distance measure for asymmetric binary variables:
$$d(i, j) = \frac{r+s}{q+r+s}$$
- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):
$$sim_{Jaccard}(i, j) = \frac{q}{q+r+s}$$
- Note: Jaccard coefficient is the same as

(a concept discussed in Pattern Discovery)

$$coherence(i, j) = \frac{sup(i, j)}{sup(i) + sup(j) - sup(i, j)} = \frac{q}{(q+r)+(q+s)-q}$$

# Example: Dissimilarity between Asymmetric Binary Variables

| Name | Gender | Fever | Cough | Test-1 | Test-2 | Test-3 | Test-4 |
|---|---|---|---|---|---|---|---|
| Jack | M | Y | N | P | N | N | N |
| Mary | F | Y | N | P | N | P | N |
| Jim | M | Y | P | N | N | N | N |

- Gender is a symmetric attribute (not counted in)
- The remaining attributes are asymmetric binary
- Let the values Y and P be 1, and the value N be 0
- Distance: $d(i, j) = \frac{r + s}{q + r + s}$

$$d(jack, mary) = \frac{0+1}{2+0+1} = 0.33$$

$$d(jack, jim) = \frac{1+1}{1+1+1} = 0.67$$

$$d(jim, mary) = \frac{1+2}{1+1+2} = 0.75$$

Jack (rows) vs. Mary (columns):

| | 1 | 0 | $\Sigma_{row}$ |
|---|---|---|---|
| 1 | 2 | 0 | 2 |
| 0 | 1 | 3 | 4 |
| $\Sigma_{col}$ | 3 | 3 | 6 |

Jack (rows) vs. Jim (columns):

| | 1 | 0 | $\Sigma_{row}$ |
|---|---|---|---|
| 1 | 1 | 1 | 2 |
| 0 | 1 | 3 | 4 |
| $\Sigma_{col}$ | 2 | 4 | 6 |

Jim (rows) vs. Mary (columns):

| | 1 | 0 | $\Sigma_{row}$ |
|---|---|---|---|
| 1 | 1 | 1 | 2 |
| 0 | 2 | 2 | 4 |
| $\Sigma_{col}$ | 3 | 3 | 6 |

# Proximity Measure for Categorical Attributes

- Categorical data, also called nominal attributes
  - Example: Color (red, yellow, blue, green), profession, etc.
- Method 1: Simple matching
  - *m*: # of matches, *p*: total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2: Use a large number of binary attributes
  - Creating a new binary attribute for each of the *M* nominal states

# Ordinal Variables

- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank (e.g., freshman, sophomore, junior, senior)
- Can be treated like interval-scaled
  - Replace *an ordinal variable value* by its rank: $r_{if} \in \{1, ..., M_f\}$
  - Map the range of each variable onto [0, 1] by replacing *i*-th object in the *f*-th variable by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

    - Example: freshman: 0; sophomore: 1/3; junior: 2/3; senior 1
      - Then distance: d(freshman, senior) = 1, d(junior, senior) = 1/3
  - Compute the dissimilarity using methods for interval-scaled variables

# Attributes of Mixed Type

- A dataset may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, and ordinal
- One may use a weighted formula to combine their effects:

$$d(i,j) = \frac{\sum_{f=1}^{p} w_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^{p} w_{ij}^{(f)}}$$

  - If $f$ is numeric: Use the normalized distance
  - If $f$ is binary or nominal: $d_{ij}^{(f)} = 0$ if $x_{if} = x_{jf}$; or $d_{ij}^{(f)} = 1$ otherwise
  - If $f$ is ordinal
    - Compute ranks $z_{if}$ (where $z_{if} = \frac{r_{if} - 1}{M_f - 1}$ )
    - Treat $z_{if}$ as interval-scaled

# Cosine Similarity of Two Vectors

- A **document** can be represented by a bag of terms or a long vector, with each attribute recording the *frequency* of a particular term (such as word, keyword, or phrase) in the document

| *Document* | *team* | *coach* | *hockey* | *baseball* | *soccer* | *penalty* | *score* | *win* | *loss* | *season* |
|---|---|---|---|---|---|---|---|---|---|---|
| Document1 | 5 | 0 | 3 | 0 | 2 | 0 | 0 | 2 | 0 | 0 |
| Document2 | 3 | 0 | 2 | 0 | 1 | 1 | 0 | 1 | 0 | 1 |
| Document3 | 0 | 7 | 0 | 2 | 1 | 0 | 0 | 3 | 0 | 0 |
| Document4 | 0 | 1 | 0 | 0 | 1 | 2 | 2 | 0 | 3 | 0 |

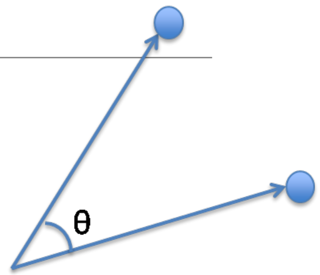
- Other vector objects: Gene features in micro-arrays
- Applications: Information retrieval, biologic taxonomy, gene feature mapping, etc.
- Cosine measure: If $d_1$ and $d_2$ are two vectors (e.g., term-frequency vectors), then

$$cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\| d_1 \| \times \| d_2 \|}$$

where • indicates vector dot product, $||d||$: the norm of vector *d*

# Example: Calculating Cosine Similarity

- Calculating Cosine Similarity: $$cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\| d_1 \| \times \| d_2 \|}$$ $$sim(A,B) = \cos(\theta) = \frac{A \cdot B}{\|A\| \|B\|}$$




  where • indicates vector dot product, $||d||$: the length of vector $d$

- Ex: Find the **similarity** between documents 1 and 2.

  $d_1$ = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0) $d_2$ = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)

  - First, calculate vector dot product

  - Then, calculate $||d_1||$ and $||d_2||$

  - Calculate cosine similarity: $\cos(d_1, d_2) =$

# Example: Calculating Cosine Similarity

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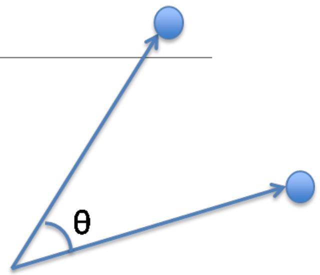


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  - First, calculate vector dot product

    $d_1 \bullet d_2$ = 5 X 3 + 0 X 0 + 3 X 2 + 0 X 0 + 2 X 1 + 0 X 1 + 0 X 1 + 2 X 1 + 0 X 0 + 0 X 1 = 25

  - Then, calculate $||d_1||$ and $||d_2||$

$$\| d_1 \| = \sqrt{5\times5+0\times0+3\times3+0\times0+2\times2+0\times0+0\times0+2\times2+0\times0+0\times0} = 6.481$$

$$\| d_2 \| = \sqrt{3\times3+0\times0+2\times2+0\times0+1\times1+1\times1+0\times0+1\times1+0\times0+1\times1} = 4.12$$

  - Calculate cosine similarity: cos($d_1$, $d_2$ ) = 25/ (6.481 X 4.12) = 0.94

# Chapter 2. Getting to Know Your Data

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

# Summary

- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing
- Many methods have been developed but still an active area of research

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